

Time Frame:	Standards:
45-60 minutes	Idaho Math Standard 1.2, 2.2, 2.3, 2.4, 3.1, 3.2
Objectives:	
<ol style="list-style-type: none"> 1. Students will be able to calculate ratios and rates. 2. Students will be able to use unit rates as a method of comparison. 3. Students will be able to convert rates. 	
Background Information:	
<ul style="list-style-type: none"> - The teacher should read through the lesson before teaching it and become familiar with the concepts covered. - The students should have a background knowledge of the following: <ul style="list-style-type: none"> o Substituting numbers for variables into equations. - This lesson uses wind turbine energy references, and thus the students and teacher should have already formed a background on wind energy. (A good way to do this is to teach the lesson Wind Power and Turbines.) 	
Materials:	
<ul style="list-style-type: none"> - Math Minutes A1.1 and Math Minutes A1.2 - Tip Speed Ratio (TSR) handout - Comparing Energy handout 	
Procedure:	
<ol style="list-style-type: none"> 1. Students complete <i>Math Minutes A1.1</i> 2. Teacher explains the definition of a ratio and the variety of ways to write them. <ul style="list-style-type: none"> As a fraction $\frac{a}{b}$ Using the word "to" a to b Using a colon $a:b$ 3. Explain the difference between ratios and fractions. (Fractions are an example of a specific type of ratio, in which the two numbers are related in a part-to-whole relationship.) 4. Explain rates, and the difference between rates and ratios. 5. Teach how to change a rate into a unit rate and how unit rates can be used as a way of comparison. (Be sure to show examples.) 6. Teach how to convert rates. (It would be good to show an example of changing miles per hour to meters per second.) 7. Handout <i>Tip Speed Ratio (TSR)</i> and <i>Comparing Energy</i> and explain worksheets to the students. 	
Vocabulary:	
Math:	
Ratio: A comparison of two numbers by division.	

Rate: A ratio comparing two different quantities.

Unit rate: A rate with a denominator of 1.

Wind Energy:

Tip Speed Ratio (TSR): ratio between the wind speed and the speed of the tips of the wind turbine blades.

Assessment:

The anticipated assessment is the completed problems from the TSR and Comparing Energy handouts, done correctly.

Also, the students will complete *Math Minutes A1.2* the following lesson day.

References:

Pre-Algebra online textbook www.pearsonsuccessnet.com

KidWind Math Documents: Tip Speed Ratio www.kidwind.org

EIA Energy Kids www.eia.doe.gov/kids

Addition Material:

Math Minutes A1.1



Name _____

Date _____

MATH MINUTES A1.1

Review

A variable is a symbol (usually a letter) that stands in place of a number. Variables are often found in equations, and they represent an unknown value. We can solve for variables to find unknown values by doing inverse operations. We can also set up an equation for a problem using variables.

Math Problems

Solve for the variable in the following equations:

1. $7 = 4 + x$
2. $18 \div y = 6$
3. $3z = 24$

Simplify the following equations for $a = 3, b = 2,$ and $c = 5$

4. $7 - c + a =$
5. $4b + b =$
6. $\frac{20}{c} - b =$

Word Problems

7. Write and solve a variable expression for the following problem:
On one windy day in April, the combined winds of 3 cities, Idaho Falls, Pocatello, and Rexburg, totaled 76 mph. If the wind in Idaho Falls was 22 mph and the wind in Rexburg was 29 mph, how fast was the wind blowing in Pocatello?

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Tip Speed Ratio (TSR) Handout:

TIP SPEED RATIO

The *Tip Speed Ratio (TSR)* is an extremely important factor in wind turbine design. TSR refers to the ratio between the wind speed and the speed of the tips of the wind turbine blades.

$$TSR (\lambda) = \frac{\text{Tip Speed of Blade}}{\text{Wind Speed}}$$

If the rotor of the wind turbine spins too slowly, most of the wind will pass straight through the gap between the blades, therefore giving it no power! But if the rotor spins too fast, the blades will blur and act like a solid wall to the wind. Also, rotor blades create turbulence as they spin through the air. If the next blade arrives too quickly, it will hit that turbulent air. So, sometimes it is actually better to *slow down* your blades!

Wind turbines must be designed with optimal tip speed ratios to get the maximum amount of power from the wind.



Before we can calculate the TSR, we need to figure out the *tip speed of blade* (which is the numerator of the TSR). To do this we need to calculate two things: distance and time.

- Distance:** The distance is how far the tip of a blade goes to make one revolution. In order to find this out we need to first know the rotor radius (length of one blade). We can then use the equation for the circumference of a circle, $C = 2\pi r$, to find the total distance traveled of the blade tip.
- Time:** We need to know how long it takes the rotor to make one full revolution. If we know the RPM (revolutions per minute) then we need to convert this unit rate into a unit rate where revolution is the denominator and time is in the numerator. There are 60 seconds in one minute, so take 60 divided by the RPM value. That will tell you how many seconds it takes to make one revolution.

Now that we know distance and time we can calculate the *tip speed of blade*:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Problems

- If you have a wind turbine with three blades, each 4 meters long, what distance does the tip of each blade travel in one full revolution?
- If this turbine is rotating at a rate of 42 Revolutions per Minute (RPM), how long does it take to make one full revolution?
- Based on your answers from 1 and 2, calculate how fast the tips of this wind turbine are moving through the air.
- If the wind is blowing at 6 meters per second, what is the tip speed ratio of this turbine? Use your answers from questions 1-3 to help solve this problem.
- According to the "optimal" tip speed ratio for this three-bladed turbine, are these blades moving too fast or too slow?
- An offshore wind turbine with three 60 meter blades rotates at a leisurely 12 RPM. The wind is whipping along at 18 meters per second. What is the tip speed ratio for this turbine? How does this compare to the "optimal" tip speed ratio for this turbine?
- A home-made turbine has blades that are 0.25 meters long. They are spinning rapidly at 600 RPM. What is the tip speed?
- You have measured the wind speed to be 9 miles per hour. How fast is the wind blowing in meters per second?
- Using the data from questions 7 and 8, calculate the Tip Speed Ratio of this wind turbine.

The only thing we have left to figure out is the *wind speed*, which is the denominator of the TSR. Wind speed is usually given in mph. However, you need to make sure that your units are all the same. For example, you can't use a wind speed in miles per hour and a tip speed in meters per second. This formula will help if you need to make some conversions:

$$1 \text{ meter per second} = 2.237 \text{ miles per hour}$$

Now that we know the tip speed of blade and the wind speed and we have converted them to the same units, we should be able to figure out the TSR!

$$TSR (\lambda) = \frac{\text{Tip Speed of Blade}}{\text{Wind Speed}}$$

If your TSR is above 1, that means there is *lift* involved to make your blades spin faster than the wind speed. If your TSR is below 1, there is a lot of drag going on! Old windmills used to lift weights, grind grain, or pump water probably had TSRs around 1. Modern wind turbines have higher TSR values like 5.

Why is This Important???

Knowing the tip speed ratio of your turbine will help you maximize the power output and efficiency of your wind turbine. Remember that if your rotor spins too slowly, a lot of wind will pass through the gaps between the blades rather than giving energy to your turbine. But if your blades spin too quickly, they could create too much turbulent air or act as a solid wall against the wind. So, if you want to maximize your turbine's efficiency, you've got to calculate the perfect Tip Speed Ratio.

How do you know the perfect tip speed Ratio?

If you want the optimum TSR for maximum power output, this formula has been empirically proven:

$$\lambda (\text{max power}) = \frac{4\pi}{n} \quad (n = \text{number of blades})$$

Answer Key

- The length of one blade is equal to the radius. The distance travelled is equal to the circumference of the circle created by the blades. Circumference of a circle = $2\pi r$. So, $(2 \times \pi \times 4) = 25.13 \text{ meters}$.
- $\frac{1 \text{ minute}}{42 \text{ revolutions}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{60}{42} = 1.43 \text{ seconds per revolution}$
- Velocity = distance divided by time. $\frac{25.13 \text{ meters}}{1.43 \text{ seconds}} = 17.6 \text{ m/s}$
- $TSR = \frac{\text{tip speed of blade}}{\text{wind speed}} = \frac{17.6}{6} = 2.93$
- Optimal $TSR = \frac{4\pi}{n} = \frac{4\pi}{3} = 4.2$ --- The blades are moving too slow
- $2 \times \pi \times 60 = 377 \text{ meters travelled}$. $\frac{60}{12} = 5 \text{ seconds per revolution}$. $\frac{377 \text{ meters}}{5 \text{ seconds}} = 75.4 \text{ m/s}$. $TSR = \frac{75.4}{18} = 4.2$ --- This is the optimal TSR for this wind turbine.
- $2 \times \pi \times 0.25 = 1.57 \text{ meters travelled}$. $600 \text{ rpm} = 10 \text{ revolutions per second}$. 1 revolution = 1.57 meters. **Tip speed = 15.7 m/s**
- 1 meter per second = 2.237 miles per hour. $9/2.237 = 4.02 \text{ m/s}$
- $TSR = \frac{\text{tip speed of blade}}{\text{wind speed}} = \frac{15.7}{4.02} = 3.91$

Comparing Energy Handout:

Name _____

Date _____

Comparing Energy

Source: <http://www.eia.doe.gov/field/energy/faq/faq-energy-unit-basics>

Background Info

Some popular units for comparing energy include British Thermal Units (Btu), barrels of oil equivalent, metric tons of oil equivalent, metric tons of coal equivalent, and terajoules.

In the United States, the Btu, a measure of heat energy, is the most commonly used unit for comparing fuels. Because energy used in different countries comes from different places, the Btu content of fuels varies slightly from country to country.

Did you know ?

A confusing unit of measure is a barrel. A barrel's capacity is determined by who uses the term and what it contains. For example:

- ✓ 1 barrel (bbl) of petroleum or related products = 42 gallons
- ✓ 1 barrel of Portland cement = 376 pounds
- ✓ 1 barrel of flour = 196 pounds
- ✓ 1 barrel of pork or fish = 200 pounds
- ✓ 1 barrel of (U.S.) dry measure = 3,291.2 bushels or 4,210.4 cubic feet
- ✓ A barrel may be called a "drum," but a drum usually holds 55 gallons
- ✓ A barrel big enough to carry a man over Niagara Falls.

Btu Content of Common Energy Units

The Btu content of each fuel provided below reflects the average energy content for fuels consumed in the United States.

Fuel	Btu Content
1 barrel (42 gallons) of crude oil	5,800,000 Btu
1 gallon of gasoline	124,000 Btu
1 gallon of diesel fuel	139,000 Btu
1 gallon of heating oil	139,000 Btu
1 barrel of residual fuel oil	6,387,000 Btu
1 cubic foot of natural gas	1,023 Btu
1 gallon of propane	91,000 Btu
1 short ton of coal	19,988,000 Btu
1 kilowatt-hour of electricity	3,412 Btu

Conversion Problems

Directions: Use the above conversion factors to answer the following problems:

- You have a natural gas furnace in your home that used 81,300 cubic feet of natural gas for heating last winter. Your neighbor has an oil furnace that used 584 gallons of heating oil last winter. Which home used more energy for heating?
- Jack owns a regular gasoline pickup, Tom owns a diesel pickup. While pulling a trailer, Jack gets on average 12 miles per gallon, and Tom gets on average 18 miles per gallon. If they both pull a trailer for 36 miles, which pickup will use more energy?
- You work for an electric power company. Your company's power generators can run on one of two fuels: natural gas or residual fuel oil. Your job is to switch fuels when the cost of the fuel you are currently using becomes more expensive than the other fuel. This will keep costs down for you and your electricity customers. Your company's generators are currently using residual fuel oil, but fuel oil prices have been going up much faster than natural gas prices. Currently, Natural Gas prices are \$7.30 per thousand cubic feet, and residual fuel oil prices are \$57.75 per barrel. Is it time to switch to natural gas?

M

Answer Key

$$1. \text{ Natural Gas: } 81,300 \frac{\text{cubic feet of natural gas}}{\text{natural gas}} \times \frac{1,023 \text{ Btu}}{1 \text{ cubic foot of natural gas}} = 83,576,400 \text{ Btu}$$

$$\text{Heating Oil: } 584 \frac{\text{gallons of heating oil}}{\text{heating oil}} \times \frac{139,000 \text{ Btu}}{1 \text{ gallon of heating oil}} = 81,176,000 \text{ Btu}$$

Answer: Your home (natural gas home) used more energy.

$$2. \text{ Jack (gas pickup): } 36 \text{ miles} \times \frac{1 \text{ gallon of gas}}{12 \text{ miles}} \times \frac{124,000 \text{ Btu}}{1 \text{ gallon of gas}} = 372,000 \text{ Btu}$$

$$\text{Tom (Diesel pickup): } 36 \text{ miles} \times \frac{1 \text{ gallon of diesel}}{18 \text{ miles}} \times \frac{139,000 \text{ Btu}}{1 \text{ gallon of diesel}} = 278,000 \text{ Btu}$$

Answer: Jack's pickup uses more energy.

$$3. \text{ Natural Gas: } \frac{\$7.30}{1,000 \text{ cubic feet}} \times \frac{1,000 \text{ cubic feet}}{1,023 \text{ million Btu}} = \$7.10/\text{million Btu}$$

$$\text{Residual Fuel Oil: } \frac{\$57.75}{1 \text{ barrel}} \times \frac{1 \text{ barrel}}{6,387 \text{ million Btu}} = \$9.19/\text{million Btu}$$

Answer: Yes, it is time to switch to residual fuel oil.

TIP SPEED RATIO

Name _____

Date _____

MATH MINUTES AL.2

Definitions

Ratio = _____

Rate = _____

Unit Rate = _____

Ratios can be expressed as _____,
_____, or _____.

Application

_____ are used as a means of comparison.

Describe at least one way rates and ratios are used in today's world: _____

Math Problems

Find each unit rate:

1. \$57 for 6 hours. _____

2. 508 rotations in 4 minutes. _____

Convert the following:

3. 3 hours = _____ seconds

4. 90 inches = _____ feet

Word Problems

Speed is a rate of distance over time. Suppose you are traveling non-stop from Idaho Falls, ID to Jackson, WY, which is approximately 97 miles. You leave Idaho Falls at 9:05 am and arrive in Jackson at 10:50 am. Also, your car gets a rate of 26 miles to the gallon. You filled it up just before you left at the cost of \$2.71/gallon.

5. What was your average rate of speed in miles per hour? _____

6. How many gallons of gas did it take you to get to Jackson? _____

7. How much did the trip cost you in gas? _____

TIP SPEED RATIO

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TIP SPEED RATIO

BLACKHAM

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$$\lambda (\text{max power}) = \frac{4\pi}{n} \quad (n = \text{number of blades})$$

Source: http://www.kidwind.org/PDFs/SUPPORT_Math_Tip%20Speed%20Ratio.pdf

Problems

1. If you have a wind turbine with three blades, each 4 meters long, what distance does the tip of each blade travel in one full revolution?
2. If this turbine is rotating at a rate of 42 Revolutions per Minute (RPM), how long does it take to make one full revolution?
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7. A home-made turbine has blades that are 0.25 meters long. They are spinning rapidly at 600 RPM. What is the tip speed?
8. You have measured the wind speed to be 9 miles per hour. How fast is the wind blowing in meters per second?
9. Using the data from questions 7 and 8, calculate the Tip Speed Ratio of this wind turbine.

Answer Key

TIP SPEED RATIO

1. The length of one blade is equal to the radius. The distance travelled is equal to the circumference of the circle created by the blades. Circumference of a circle = $2\Pi r$. So, $(2 \times \Pi \times 4) = \mathbf{25.13 \text{ meters}}$.
2. $\frac{1 \text{ minute}}{42 \text{ revolutions}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{60}{42} = \mathbf{1.43 \text{ seconds per revolution}}$
3. Velocity = distance divided by time. = $\frac{25.13 \text{ meters}}{1.43 \text{ seconds}} = \mathbf{17.6 \text{ m/s}}$
4. $\text{TSR} = \frac{\text{tip speed of blade}}{\text{wind speed}} = \frac{17.6}{6} = \mathbf{2.93}$
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 $\frac{377 \text{ meters}}{5 \text{ seconds}} = 75.4 \text{ m/s}$. $\text{TSR} = 75.4/18 = \mathbf{4.2}$ --- This is the optimal TSR for this wind turbine.
7. $2 \times \Pi \times 0.25 = 1.57 \text{ meters travelled}$. $600 \text{ rpm} = 10 \text{ revolutions per second}$. 1 revolution = 1.57 meters. **Tip speed = 15.7 m/s**
8. 1 meter per second = 2.237 miles per hour. $9/2.237 = \mathbf{4.02 \text{ m/s}}$
9. $\text{TSR} = \frac{\text{tip speed of blade}}{\text{wind speed}} = \frac{15.7}{4.02} = \mathbf{3.91}$

Comparing Energy

Background Info	Did you know ?
<p>Some popular units for comparing energy include British Thermal Units (Btu), barrels of oil equivalent, metric tons of oil equivalent, metric tons of coal equivalent, and terajoules.</p> <p>In the United States, the Btu, a measure of heat energy, is the most commonly used unit for comparing fuels. Because energy used in different countries comes from different places, the Btu content of fuels varies slightly from country to country.</p>	<p>A confusing unit of measure is a barrel. A barrel's capacity is determined by who uses the term and what it contains. For example:</p> <ul style="list-style-type: none"> ✓ 1 barrel (bbl) of petroleum or related products = 42 gallons ✓ 1 barrel of Portland cement = 376 pounds ✓ 1 barrel of flour = 196 pounds ✓ 1 barrel of pork or fish = 200 pounds ✓ 1 barrel of (U.S.) dry measure = 3.29122 bushels or 4.2104 cubic feet ✓ A barrel may be called a "drum," but a drum usually holds 55 gallons ✓ A barrel big enough to carry a man over Niagara Falls.

Btu Content of Common Energy Units

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1 gallon of propane	91,000 Btu
1 short ton of coal	19,988,000 Btu
1 kilowatthour of electricity	3,412 Btu

Source: http://www.eia.doe.gov/kids/energy.cfm?page=about_energy_units-basics

Answer Key

1. Natural Gas:

$$81,300 \text{ ~~cubic ft of natural gas~~} \times \frac{1,028 \text{ Btu}}{1 \text{ cubic ft of natural gas}} = 83,576,400 \text{ Btu}$$

Heating Oil: $584 \text{ ~~gallons of heating oil~~} \times \frac{139,000 \text{ Btu}}{1 \text{ gallon of heating oil}} = 81,176,000 \text{ Btu}$

Answer: Your home (natural gas home) used more energy.

2. Jack (gas pickup): $36 \text{ miles} \times \frac{1 \text{ gallon of gas}}{12 \text{ miles}} \times \frac{124,000 \text{ Btu}}{1 \text{ gallon of gas}} = 372,000 \text{ Btu}$

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Answer: Jack's pickup uses more energy.

3. Natural Gas: $\frac{\$7.30}{1,000 \text{ cubic feet}} \times \frac{1000 \text{ cubic feet}}{1.028 \text{ million Btu}} = \$7.10/\text{million Btu}$

Residual Fuel Oil: $\frac{\$57.75}{1 \text{ barrel}} \times \frac{1 \text{ barrel}}{6.287 \text{ million Btu}} = \$9.19/\text{million Btu}$

Answer: Yes, it is time to switch to natural gas.